Consult the Scholar

The Role of Artificial Intelligence in Theorem Proving

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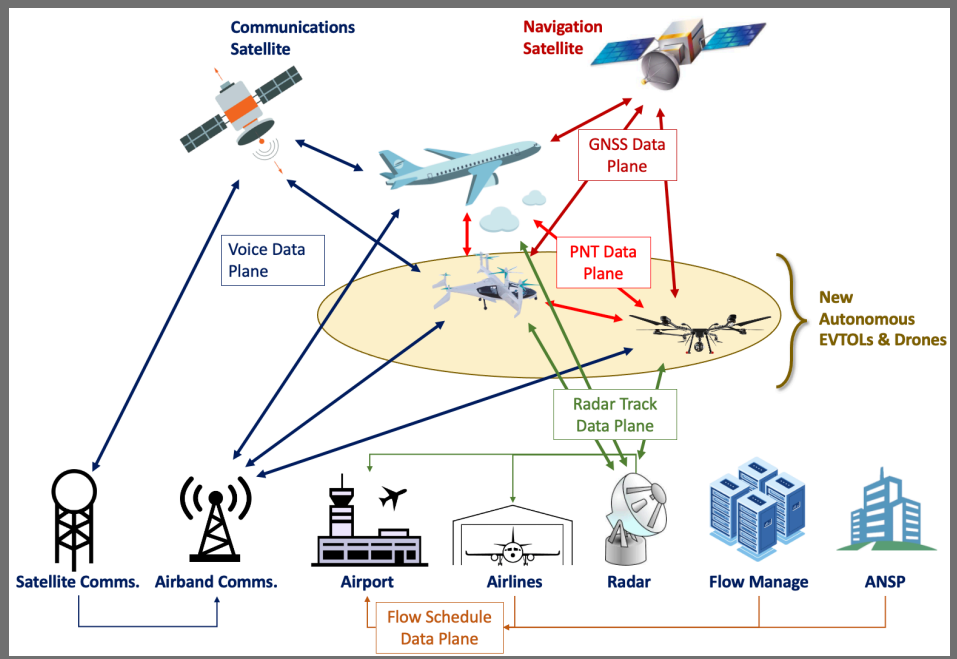
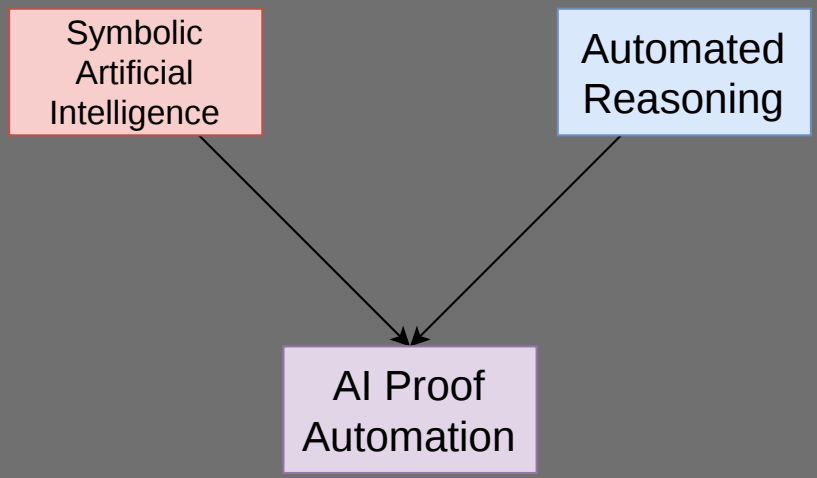
Math is Everywhere

- All modern infrastructure uses advanced math
- Arguably impossible without *automation*
  - Predicting the behavior of complex systems? Linear Algebra!
  - Arranging groups of things? Combinatorics!



The "Ultimate" Automation

- AI is everywhere
- Useful automation tool because it learns the rules of a system
- No surprise: we're interested in applying AI to modern math



The "Ultimate" Automation

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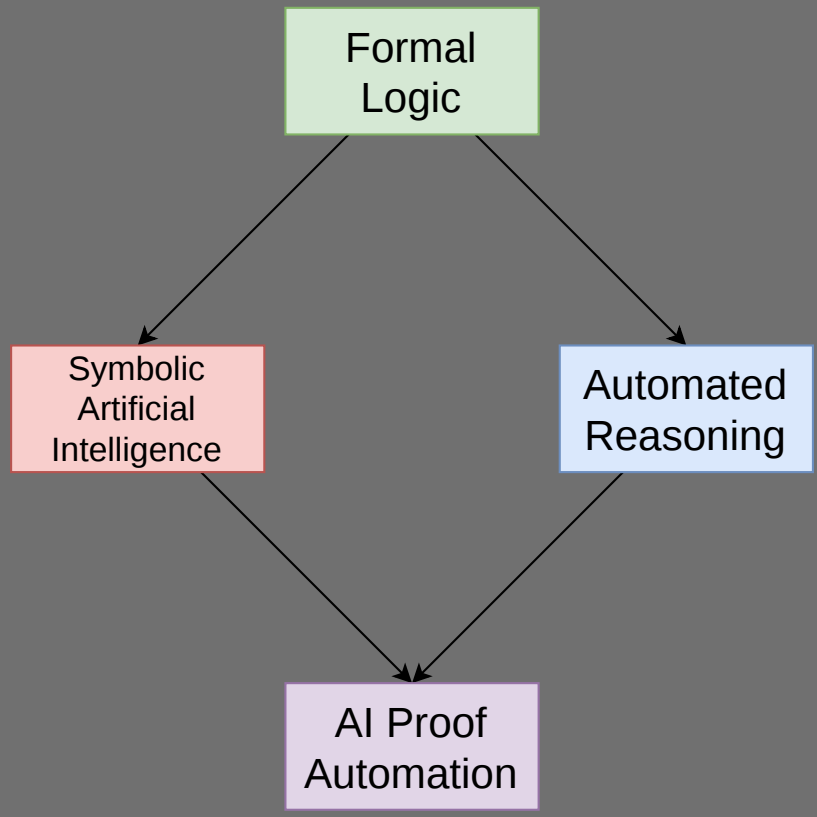
- No surprise: we're interested in applying AI to modern math
- Yes surprise: AI and mechanized math used to be deeply intertwined!



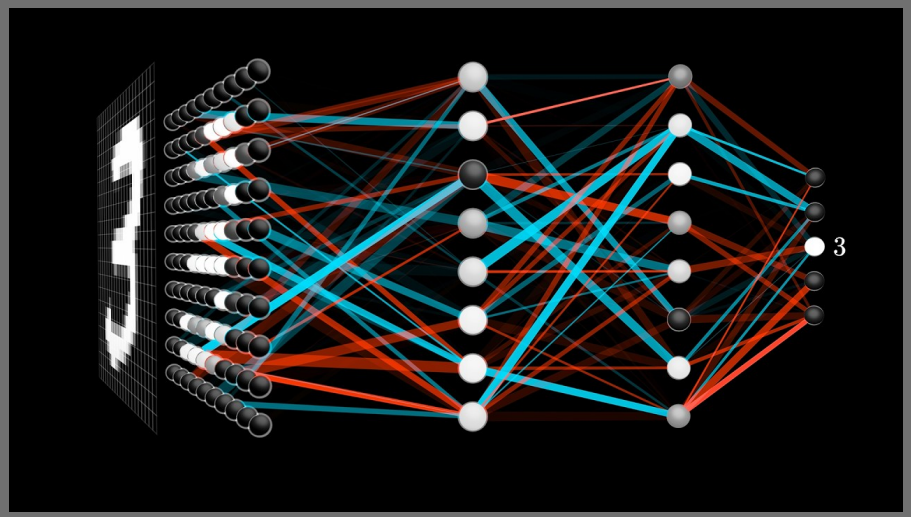
Mechanized Math?

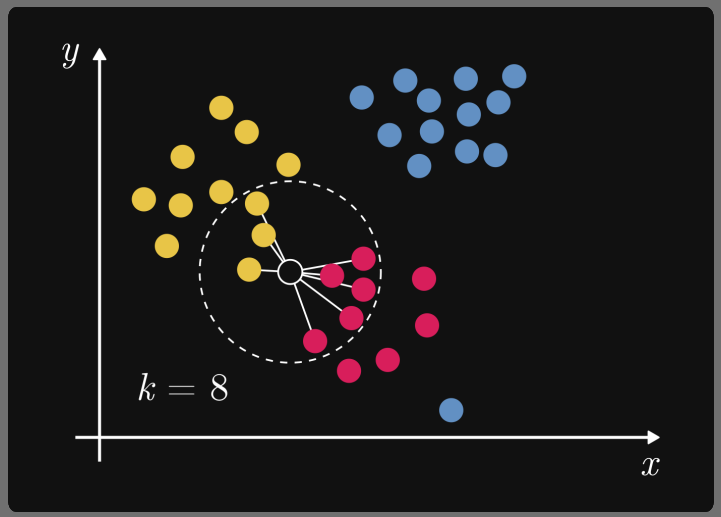
- *Proof assistants*: programs that check the validity of proofs
- Extremely trustworthy by construction
- Started becoming mainstream approx. 10 years ago
- CompCert, sel4, 4 color thm, Kepler conjecture


```
1 goal

  n : nat
  ============================
  n + 0 = n

add_0_r < induction n.
2 goals

  ============================
  0 + 0 = 0

goal 2 is:
 S n + 0 = S n
```

Machine Learning

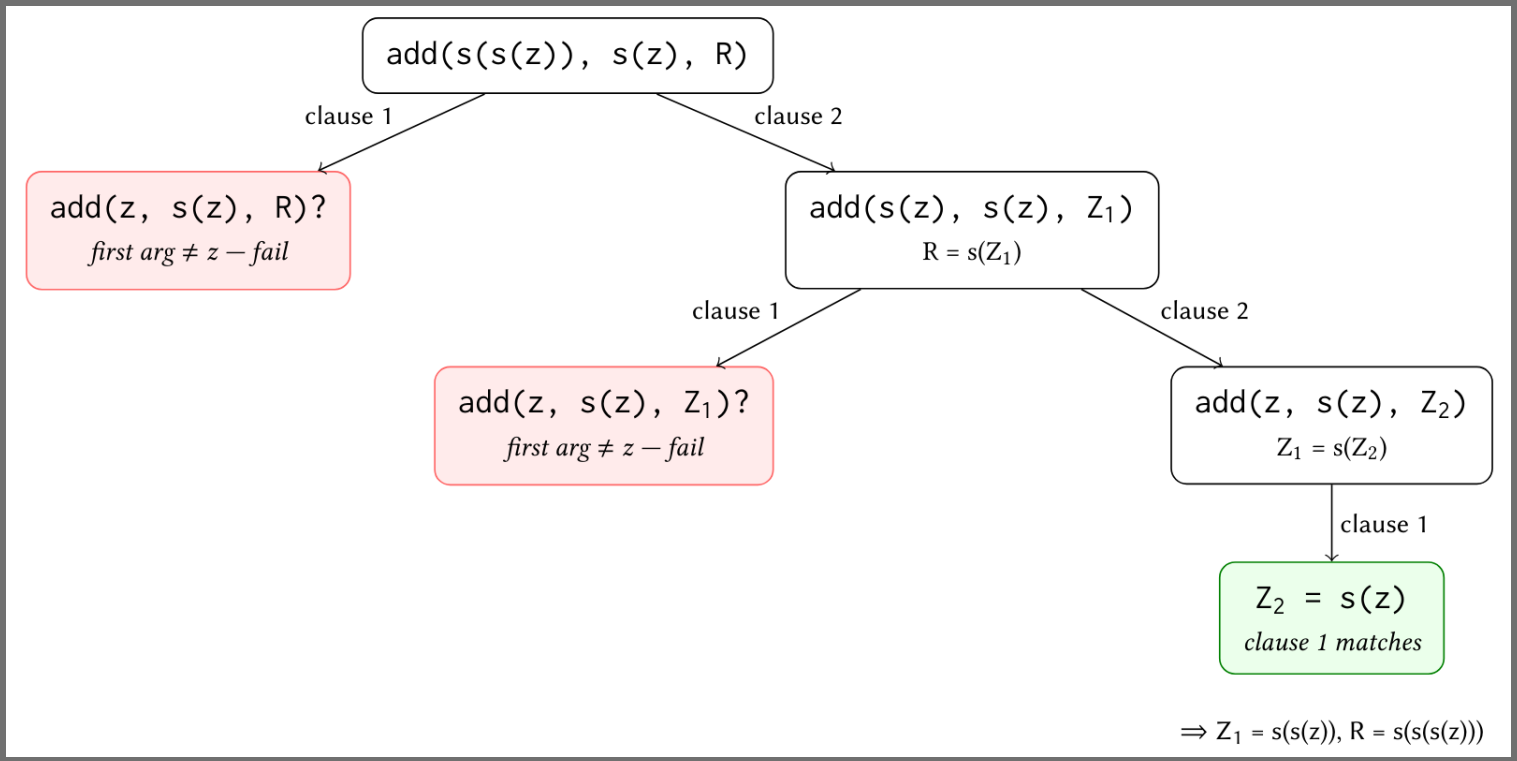
- Class of techniques for learning the rules of a system
- *Neural Network*: groundbreaking generic ML approach capable of learning deep relationships
- NNs can be assembled to fit the need, e.g., *Large Language Models*





Rule-Based Automation

- Encode reasoning strategies in deterministic algorithms
- No training required
- Predictable, suited for specific tasks
- Can be guaranteed to find answers



Proof Automation Tasks

Four primary tasks make up AI proof automation research:

- **Tactic Prediction**: deciding what sequence of reasoning steps to make in a proof
- **Premise Selection**: deciding which known facts are most useful for a specific proof
- **Theory Exploration**: choosing which theorems to prove
- **Autoformalization**: translating natural-language theorems and proofs to proof assistants

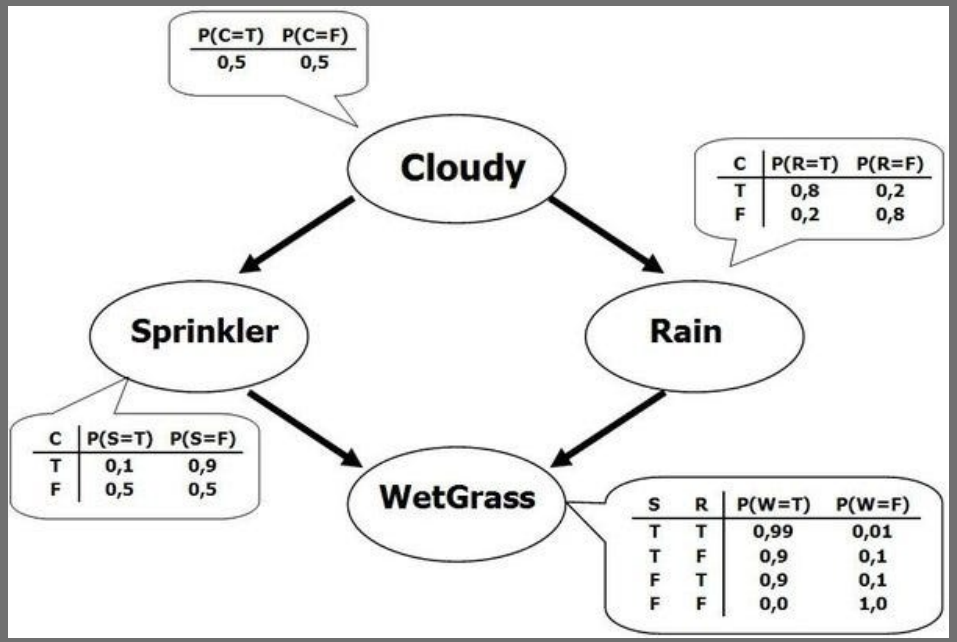
Classical Machine Learning

MaLARea (2007): first case of modern ML for proof automation

- Bayes classifier, takes theorem symbols as input, gives list of premises as output
- Achieved ~50% success on MPTP challenge, ~double that of SOTA

TacticToe (2018): k-NN for premise selection and tactic prediction

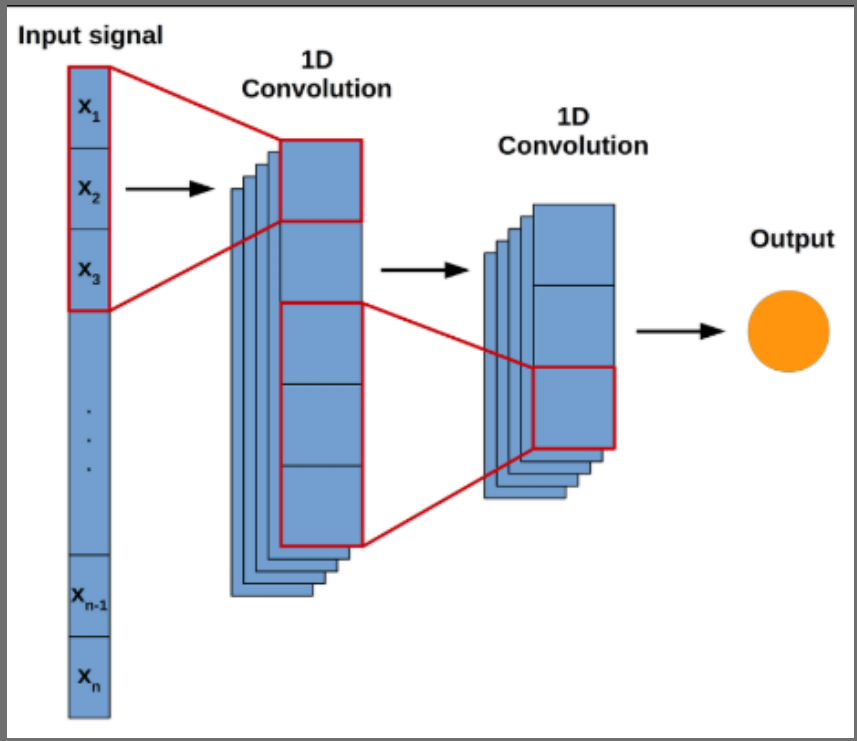
- Proves ~70% of HOL Light stdlib in < 1 min
- Ensemble: MCTS with k-NN heuristics

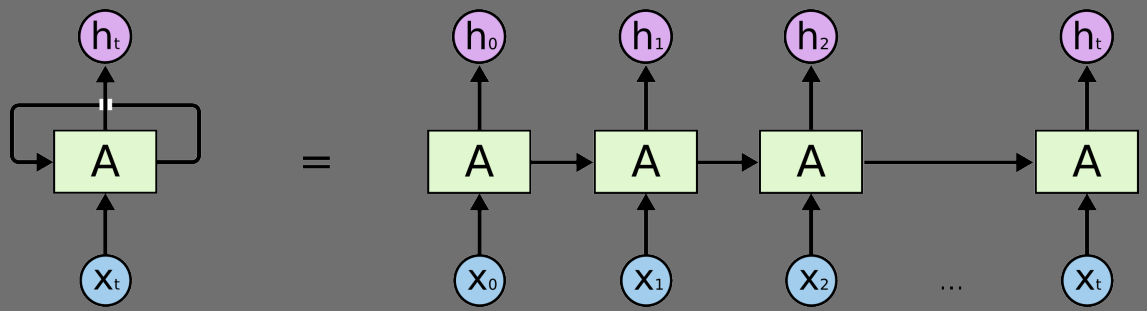


Deep Learning

DeepMath (2016): first case of DL for proof automation

- Evaluated CNNs, LSTMs, GRUs on premise selection for MML
- CNNs perform best (~70% of thms provable by ATPs) - local features most important

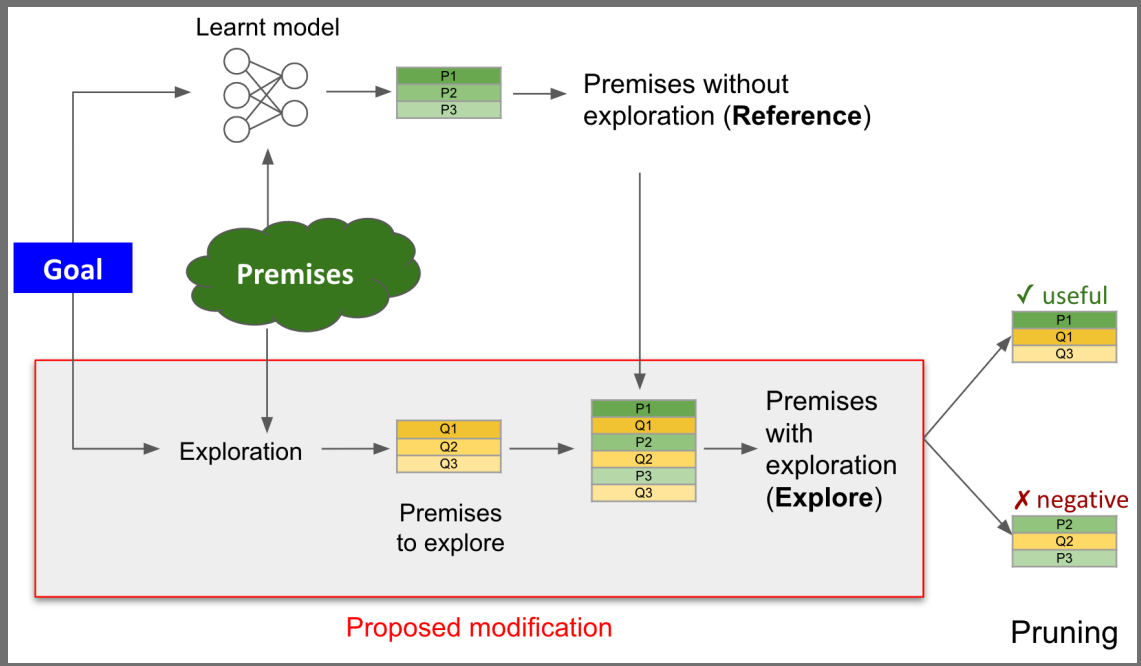




Deep Learning

Learning to Reason in Large Theories Without Imitation (2019)

- Problem: models work best when inputs are close to training data, but they should work on unseen data (hard proofs)
- Idea: *retrieval-augmented* reinforcement learning
- Find relevant premises via similarity metric to supplant model predictions → more RL training data



Large Language Models

GPT-f: first LLM proof automation project

- Takes in *whole proof history* to predict next token
- Found better results by fine-tuning on mathematical text

LeanDojo: Environment for Lean models

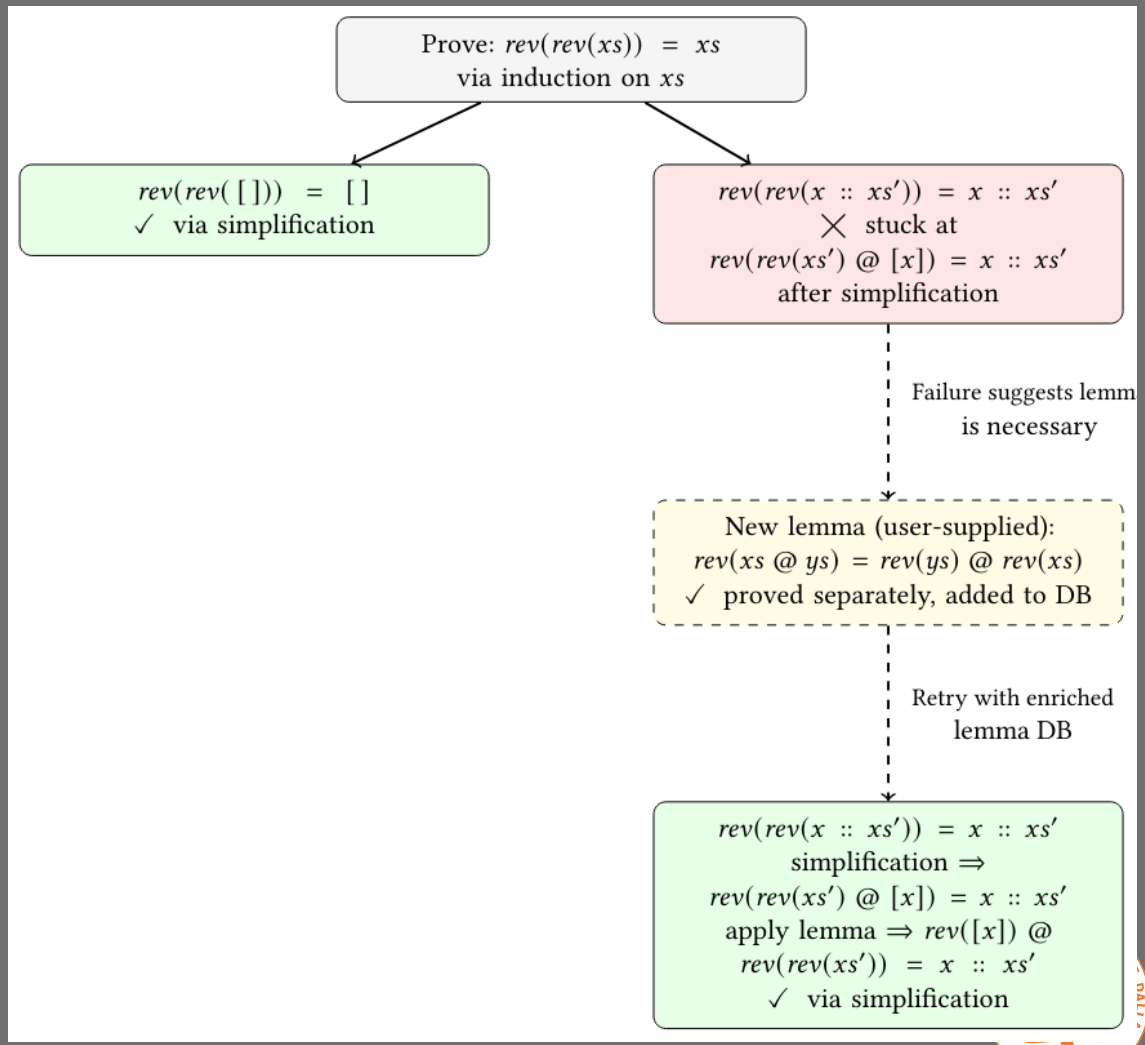
- Identified critical training issue in all modern DL approaches
- Proposes workarounds via artificial diversity measures

Draft, Sketch, Prove

- Problem: translation gap between prompt "prove this theorem" and actual proof script
- Solution: ask LLM to *draft* an informal proof, *sketch* a high-level formal proof, then dispatch to ATP

Honorable Mention - Inductive Automation

- *Nqthm/ACL2*: prominent ATP driven by heuristics
- Heuristics based around simplifying induction/recursion to make proof progress
- *Rippling*: generic inductive automation technique for simplifying inductive goals



Evaluation Metrics

```
Rocq < Search (_ + _).
plus_O_n: forall n : nat, 0 + n = n
plus_n_O: forall n : nat, n = n + 0
plus_n_Sm: forall n m : nat, S (n + m) = n + S m
plus_Sn_m: forall n m : nat, S n + m = S (n + m)
mult_n_Sm: forall n m : nat, n * m + n = n * S m
```

- Notice anything wrong?
- Distinct theorems with nearly-identical statements in train + test sets
- LeanDojo points this out, suggests fix, but incomplete
- What about similar proofs?

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Evaluation Metrics

```
Ltac solveh help :=
  intros;
  let IHx' := fresh "IH" in
  multimatch goal with
  | [x : nat |- _] => induction x as [| x' IHx']
  | [b : bool |- _] => destruct b
  | [b : bin |- _] => induction b as [| x' IHx']
  end;
  [simpl; now try rewrite help
  | simpl in *; try easy; try (rewrite help || rewrite <- help); rewrite IHx';
    try (rewrite help); try easy;
    match goal with | [|- ?G] => idtac G end].
Ltac solve := solveh eq_sym.
```

- This (rudimentary) proof procedure proves **70%** of the theorems in one chapter of Software Foundations
- Proofs can be similar too!
- If models memorize proof patterns that can be trivially automated, is that success?

Autoformalization and Theory Exploration

Large-scale autoformalization considered ∼impossible right now

- No way to automate data generation
- No way to automatically evaluate
- Arguably a subjective task

Theory exploration is the opposite of what prediction models do!

- Standard LLMs will not generate interesting theorems without substantial third-party input
- Maybe RL? Genetic algorithms?

Dataset Generation

- Python code makes up ~20% of all code on GitHub (approx. 84 million repositories)
- There are almost as many Python contributors as there are stars for the Rocq compiler
- Rocq is 7 months older than Python

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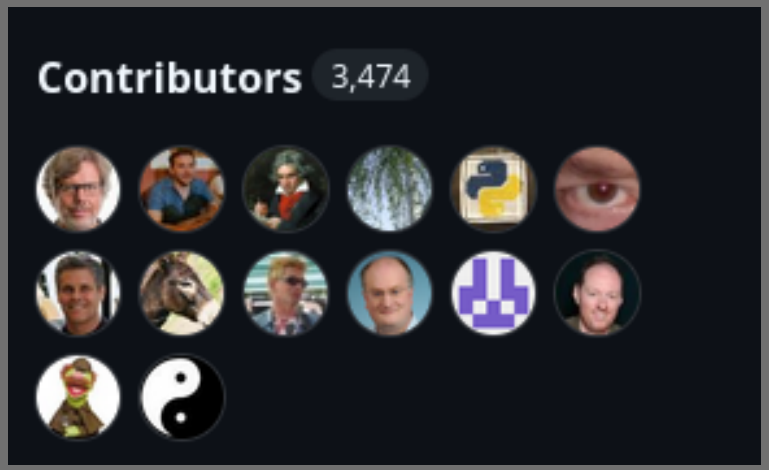

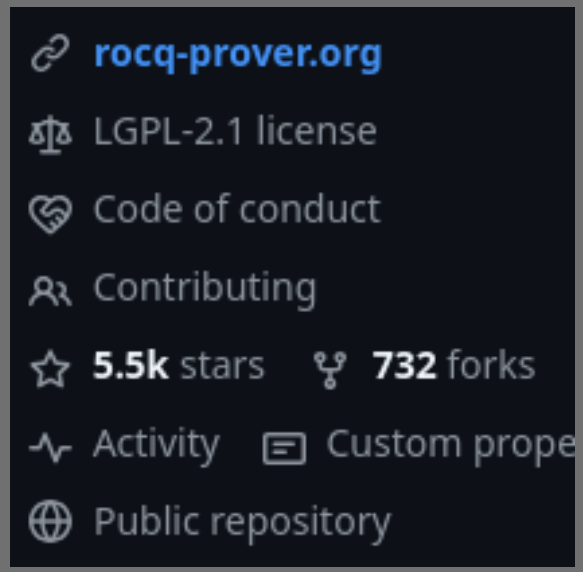

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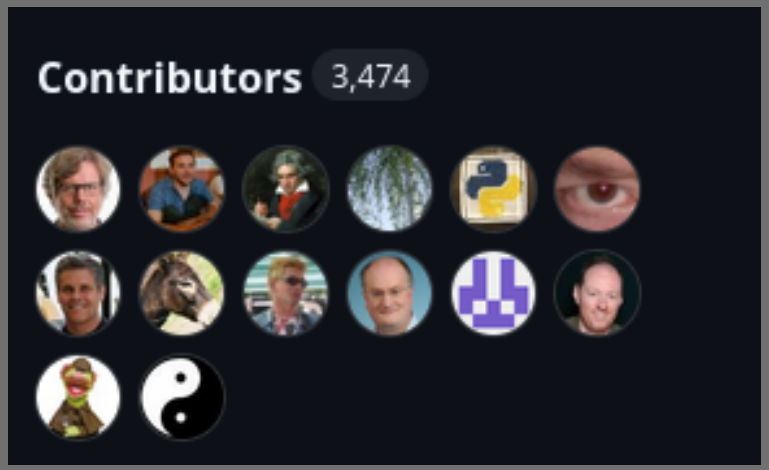

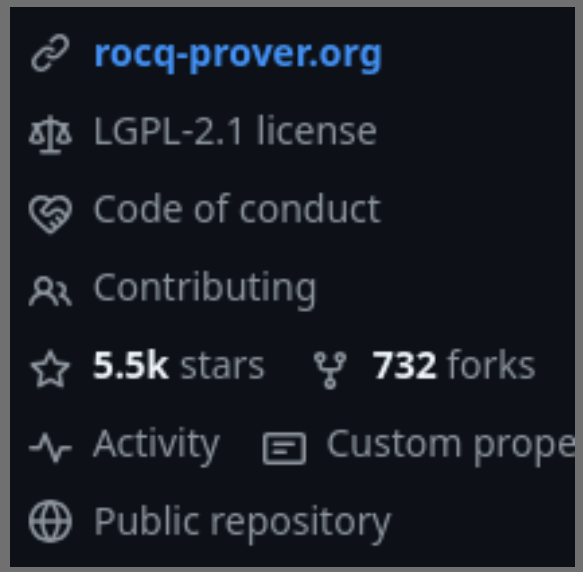

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- Survey: `https://www.charles.systems/writings/qe.pdf`
- Dependency graph: `https://www.charles.systems/qe`