Secrets of the Universe

The Ultimate Formal Verification Talk

Charles Averill

UTD Computer Security Group
The University of Texas at Dallas

Fall 2024

The Philosophy of Uncertainty

"*The only true wisdom is in knowing you know nothing.*" - Socrates

What do we know?

- Start simple, things like:

What do we know?

- Start simple, things like:
  - $3 + 4 = 6 + 1$

What do we know?

- Start simple, things like:
  - `3 + 4 = 6 + 1`
  - `red + blue = purple`

What do we know?

- Start simple, things like:
  - `3 + 4 = 6 + 1`
  - `red + blue = purple`
  - The sky is blue

What do we know?

- Start simple, things like:
  - `3 + 4 = 6 + 1`
  - `red + blue = purple`
  - The sky is blue
- Maybe we know simple algebraic relations:

What do we know?

- Start simple, things like:
  - `3 + 4 = 6 + 1`
  - `red + blue = purple`
  - The sky is blue
- Maybe we know simple algebraic relations:
  - `If y = 0, then x + y = x`

What do we know?

- Start simple, things like:
  - `3 + 4 = 6 + 1`
  - `red + blue = purple`
  - The sky is blue
- Maybe we know simple algebraic relations:
  - `If y = 0, then x + y = x`
  - $0 < n \Rightarrow \sqrt{n^2} = n$

What do we know?

- Start simple, things like:
  - `3 + 4 = 6 + 1`
  - `red + blue = purple`
  - The sky is blue
- Maybe we know simple algebraic relations:
  - `If y = 0, then x + y = x`
  - $0 < n \Rightarrow \sqrt{n^2} = n$
- Maybe we've even studied some more complex things:

What do we know?

- Start simple, things like:
  - `3 + 4 = 6 + 1`
  - `red + blue = purple`
  - The sky is blue
- Maybe we know simple algebraic relations:
  - `If y = 0, then x + y = x`
  - $0 < n \Rightarrow \sqrt{n^2} = n$
- Maybe we've even studied some more complex things:
  - Supply-Demand curves

What do we know?

- Start simple, things like:
  - `3 + 4 = 6 + 1`
  - `red + blue = purple`
  - The sky is blue
- Maybe we know simple algebraic relations:
  - `If y = 0, then x + y = x`
  - $0 < n \Rightarrow \sqrt{n^2} = n$
- Maybe we've even studied some more complex things:
  - Supply-Demand curves
  - Human Psychology

What do we know?

- Start simple, things like:
  - `3 + 4 = 6 + 1`
  - `red + blue = purple`
  - The sky is blue
- Maybe we know simple algebraic relations:
  - `If y = 0, then x + y = x`
  - $0 < n \Rightarrow \sqrt{n^2} = n$
- Maybe we've even studied some more complex things:
  - Supply-Demand curves
  - Human Psychology
  - General Relativity

Uncertainty

- Life is inherently filled with uncertainty

Uncertainty

- Life is inherently filled with uncertainty
- Humans have evolved to handle uncertainty by guessing, and we have become really good at it

Uncertainty

- Life is inherently filled with uncertainty
- Humans have evolved to handle uncertainty by guessing, and we have become really good at it
- We only have to make a few important guesses, then combine them to get cool results

Uncertainty

- Life is inherently filled with uncertainty
- Humans have evolved to handle uncertainty by guessing, and we have become really good at it
- We only have to make a few important guesses, then combine them to get cool results

Uncertainty

- Life is inherently filled with uncertainty
- Humans have evolved to handle uncertainty by guessing, and we have become really good at it
- We only have to make a few important guesses, then combine them to get cool results - this is called math
- Unfortunately, math takes too long, so we add extra guesses to the mix - these extras are often incorrect

Uncertainty

- “Certainty traps”, where we are certain we’re right with no evidence, are the main cause for things like

Uncertainty

- “Certainty traps”, where we are certain we’re right with no evidence, are the main cause for things like
  - Database breaches

Uncertainty

- “Certainty traps”, where we are certain we’re right with no evidence, are the main cause for things like
    - Database breaches
    - Physical infrastructure failures

Uncertainty

- "Certainty traps", where we are certain we're right with no evidence, are the main cause for things like
  - Database breaches
  - Physical infrastructure failures
  - Poor public policy

Uncertainty

- “Certainty traps”, where we are certain we’re right with no evidence, are the main cause for things like
  - Database breaches
  - Physical infrastructure failures
  - Poor public policy
  - Eating bad food at a restaurant

Uncertainty

- “Certainty traps”, where we are certain we’re right with no evidence, are the main cause for things like
  - Database breaches
  - Physical infrastructure failures
  - Poor public policy
  - Eating bad food at a restaurant
- We’ve developed more guesses and checks to mitigate failures like these, but they are not 100% effective

Certainty in Security

"*HIC MANEBIMVS OPTIME*" - Marcus Furius Camillus

Hope

- We have established that we live in a low-certainty world

Pros	Cons
Enhanced security	Expensive
High reliability	Takes a long time
Less maintenance	Significantly more difficult
High trustworthiness	Hard to scale

Hope

- We have established that we live in a low-certainty world
- This is not the end: enter High-Assurance Computing

Pros	Cons
Enhanced security	Expensive
High reliability	Takes a long time
Less maintenance	Significantly more difficult
High trustworthiness	Hard to scale

Hope

- We have established that we live in a low-certainty world
- This is not the end: enter High-Assurance Computing
- “Let’s make rigorous, mathematically-defined checkable models of computing so we can verify that we made the software correctly”

Pros	Cons
Enhanced security	Expensive
High reliability	Takes a long time
Less maintenance	Significantly more difficult
High trustworthiness	Hard to scale

Dumb Bugs

- Anecdotally, it is clear that we as a species are very good at being *engineers* - architecting large projects to solve complex problems

Dumb Bugs

- Anecdotally, it is clear that we as a species are very good at being *engineers* - architecting large projects to solve complex problems
- Anecdotally, it is clear that we as a species are really terrible when it comes to making mistakes - they are everywhere, and they are critical

Dumb Bugs

- Anecdotally, it is clear that we as a species are very good at being *engineers* - architecting large projects to solve complex problems
- Anecdotally, it is clear that we as a species are really terrible when it comes to making mistakes - they are everywhere, and they are critical
- This is perhaps the most clear in the realm of software

Dumb Bugs

- Anecdotally, it is clear that we as a species are very good at being *engineers* - architecting large projects to solve complex problems
- Anecdotally, it is clear that we as a species are really terrible when it comes to making mistakes - they are everywhere, and they are critical
- This is perhaps the most clear in the realm of software
- Over the past 25 years, increased interest in cybersecurity led to scrutiny in our cathedrals and public works

Dumb Bugs

- Anecdotally, it is clear that we as a species are very good at being *engineers* - architecting large projects to solve complex problems
- Anecdotally, it is clear that we as a species are really terrible when it comes to making mistakes - they are everywhere, and they are critical
- This is perhaps the most clear in the realm of software
- Over the past 25 years, increased interest in cybersecurity led to scrutiny in our cathedrals and public works
- Countless "dumb bugs," tiny one-line mistakes that turn an invaluable utility into a weapon, have been discovered

Case Study

- Many examples of "dumb" software bugs with huge impacts

Case Study

- Many examples of “dumb” software bugs with huge impacts
- Heartbleed (2014): Buffer overflow in assumed benign code allows for huge information leakage

Case Study

- Many examples of “dumb” software bugs with huge impacts
- Heartbleed (2014): Buffer overflow in assumed benign code allows for huge information leakage
- Shellshock (2014): Unjustified trust in environment variables allows for RCE on the majority of online systems

Case Study

- Many examples of "dumb" software bugs with huge impacts
- Heartbleed (2014): Buffer overflow in assumed benign code allows for huge information leakage
- Shellshock (2014): Unjustified trust in environment variables allows for RCE on the majority of online systems
- Spectre/Meltdown (2018): Unjustified trust in information-concealing properties of speculative execution allows for huge information leakage

Case Study

- Many examples of "dumb" software bugs with huge impacts
- Heartbleed (2014): Buffer overflow in assumed benign code allows for huge information leakage
- Shellshock (2014): Unjustified trust in environment variables allows for RCE on the majority of online systems
- Spectre/Meltdown (2018): Unjustified trust in information-concealing properties of speculative execution allows for huge information leakage
- BlueBorne (2017): Let's find out!

BlueBorne

- Collection of 8 cross-platform vulnerabilities in the Bluetooth stack

```c
static int l2cap_parse_conf_rsp(struct l2cap_chan *chan, void *rsp, int len,
                void *data, u16 *result)
{
    struct l2cap_conf_req *req = data;
    void *ptr = req->data;
    // ...
    while (len >= L2CAP_CONF_OPT_SIZE) {
        len -= l2cap_get_conf_opt(&rsp, &type, &olen, &val);

        switch (type) {
        case L2CAP_CONF_MTU:
            // Validate MTU...
            l2cap_add_conf_opt(&ptr, L2CAP_CONF_MTU, 2, chan->imtu);
            break;

        case L2CAP_CONF_FLUSH_TO:
            chan->flush_to = val;
            l2cap_add_conf_opt(&ptr, L2CAP_CONF_FLUSH_TO,
                       2, chan->flush_to);
            break;

        // ...
        }
    }
    // ...
    return ptr - data;
}
```

Excerpt from *l2cap_parse_conf_rsp* (net/bluetooth/l2cap_core.c)

BlueBorne

- Collection of 8 cross-platform vulnerabilities in the Bluetooth stack
- One of the Linux vulnerabilities - This one allows for RCE

```c
static int l2cap_parse_conf_rsp(struct l2cap_chan *chan, void *rsp, int len,
                void *data, u16 *result)
{
    struct l2cap_conf_req *req = data;
    void *ptr = req->data;
    // ...
    while (len >= L2CAP_CONF_OPT_SIZE) {
        len -= l2cap_get_conf_opt(&rsp, &type, &olen, &val);

        switch (type) {
        case L2CAP_CONF_MTU:
            // Validate MTU...
            l2cap_add_conf_opt(&ptr, L2CAP_CONF_MTU, 2, chan->imtu);
            break;

        case L2CAP_CONF_FLUSH_TO:
            chan->flush_to = val;
            l2cap_add_conf_opt(&ptr, L2CAP_CONF_FLUSH_TO,
                        2, chan->flush_to);
            break;

        // ...
        }
    }
    // ...
    return ptr - data;
}
```

Excerpt from *l2cap_parse_conf_rsp* (net/bluetooth/l2cap_core.c)

BlueBorne

`rsp` is an attacker-controlled buffer, this function intends to parse `rsp` as a list of items via `l2cap_get_conf_opt`, validate it, and copy it into `data`. Do you see the issue?

```
static int l2cap_parse_conf_rsp(struct l2cap_chan *chan, void *rsp, int len,
                void *data, u16 *result)
{
    struct l2cap_conf_req *req = data;
    void *ptr = req->data;
    // ...
    while (len >= L2CAP_CONF_OPT_SIZE) {
        len -= l2cap_get_conf_opt(&rsp, &type, &olen, &val);

        switch (type) {
        case L2CAP_CONF_MTU:
            // Validate MTU...
            l2cap_add_conf_opt(&ptr, L2CAP_CONF_MTU, 2, chan->imtu);
            break;

        case L2CAP_CONF_FLUSH_TO:
            chan->flush_to = val;
            l2cap_add_conf_opt(&ptr, L2CAP_CONF_FLUSH_TO,
                           2, chan->flush_to);
            break;

        // ...
        }
    }
    // ...
    return ptr - data;
}
```

Excerpt from *l2cap_parse_conf_rsp* (net/bluetooth/l2cap_core.c)

BlueBorne

The size of the `data` buffer isn't taken into account! A payload can be crafted in `rsp` that overflows `data` and writes arbitrary data into memory.

```
static int l2cap_parse_conf_rsp(struct l2cap_chan *chan, void *rsp, int len,
                void *data, u16 *result)
{
    struct l2cap_conf_req *req = data;
    void *ptr = req->data;
    // ...
    while (len >= L2CAP_CONF_OPT_SIZE) {
        len -= l2cap_get_conf_opt(&rsp, &type, &olen, &val);

        switch (type) {
        case L2CAP_CONF_MTU:
            // Validate MTU...
            l2cap_add_conf_opt(&ptr, L2CAP_CONF_MTU, 2, chan->imtu);
            break;

        case L2CAP_CONF_FLUSH_TO:
            chan->flush_to = val;
            l2cap_add_conf_opt(&ptr, L2CAP_CONF_FLUSH_TO,
                       2, chan->flush_to);
            break;

        // ...
        }
    }
    // ...
    return ptr - data;
}
```

Excerpt from *l2cap_parse_conf_rsp* (net/bluetooth/l2cap_core.c)

BlueBorne

- Can usually be triggered without any user interaction due to some exfiltration techniques overlooked by the Bluetooth specification

BlueBorne

- Can usually be triggered without any user interaction due to some exfiltration techniques overlooked by the Bluetooth specification
- Most BT devices are always listening for traffic directed to them, and only a hardware address is needed to initiate connection

BlueBorne

- Can usually be triggered without any user interaction due to some exfiltration techniques overlooked by the Bluetooth specification
- Most BT devices are always listening for traffic directed to them, and only a hardware address is needed to initiate connection
- 5.3b devices at risk at time of discovery, 2b after 1 year

BlueBorne

- Can usually be triggered without any user interaction due to some exfiltration techniques overlooked by the Bluetooth specification
- Most BT devices are always listening for traffic directed to them, and only a hardware address is needed to initiate connection
- 5.3b devices at risk at time of discovery, 2b after 1 year
- Hardware address supposed to be difficult to find, but actually fairly easy if you can sniff BT packets due to plenty of un-encrypted header data

BlueBorne

- Can usually be triggered without any user interaction due to some exfiltration techniques overlooked by the Bluetooth specification
- Most BT devices are always listening for traffic directed to them, and only a hardware address is needed to initiate connection
- 5.3b devices at risk at time of discovery, 2b after 1 year
- Hardware address supposed to be difficult to find, but actually fairly easy if you can sniff BT packets due to plenty of un-encrypted header data
- Stack overflows like this one usually mitigated by stack protection techniques - many Linux devices don't use these by default

BlueBorne

- Can usually be triggered without any user interaction due to some exfiltration techniques overlooked by the Bluetooth specification
- Most BT devices are always listening for traffic directed to them, and only a hardware address is needed to initiate connection
- 5.3b devices at risk at time of discovery, 2b after 1 year
- Hardware address supposed to be difficult to find, but actually fairly easy if you can sniff BT packets due to plenty of un-encrypted header data
- Stack overflows like this one usually mitigated by stack protection techniques - many Linux devices don't use these by default
- If standard mitigations don't work, what does?

Formal Verification of Simple Memory

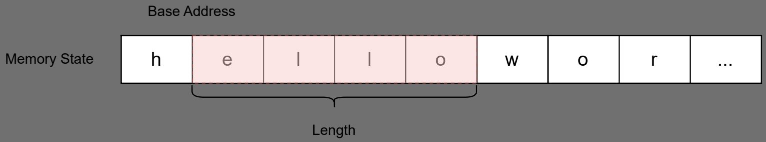
Memory is a function:

$$mem(x) = \text{contents of memory at position } x$$
$$set(mem, x, data) = \text{new memory state with } data \text{ at position } x$$

h	e	l	l	o	w	o	r	...

Arrays are a high-level construct that live on top of memory:

$$array = \{\text{memory state; base array address; array length}\}$$



Formal Verification of Simple Memory

Accessing, writing data in an array:

$$array.mem(array.base\_addr + index)$$
$$set(array.mem, array.base\_addr + index, data)$$

Is this safe?

Formal Verification of Simple Memory

Accessing, writing data in an array:

$$array.mem(array.base\_addr + index)$$
$$set(array.mem, array.base\_addr + index, data)$$

Is this safe? No!

Formal Verification of Simple Memory

Accessing, writing data in an array:

$$array.mem(array.base\_addr + index)$$
$$set(array.mem, array.base\_addr + index, data)$$

Is this safe? No!

Proposed safe accesses and writes:

$$array.mem(array.base\_addr + (index \bmod array.size)$$
$$set(array.mem, (array.base\_addr + (index \bmod array.size, data)$$

Formal Verification of Simple Memory

Accessing, writing data in an array:

$$array.mem(array.base\_addr + index)$$
$$set(array.mem, array.base\_addr + index, data)$$

Is this safe? No!

Proposed safe accesses and writes:

$$array.mem(array.base\_addr + (index \bmod array.size)$$
$$set(array.mem, (array.base\_addr + (index \bmod array.size, data)$$

Can we prove this? Yes!

Formal Verification of Simple Memory

Accessing, writing data in an array:

$$array.mem(array.base\_addr + index)$$
$$set(array.mem, array.base\_addr + index, data)$$

Is this safe? No!

Proposed safe accesses and writes:

$$array.mem(array.base\_addr + (index \bmod array.size)$$
$$set(array.mem, (array.base\_addr + (index \bmod array.size, data)$$

Can we prove this? Yes!

Should you trust me/the system?

Formal Verification of Simple Memory

Accessing, writing data in an array:

$$array.mem(array.base\_addr + index)$$
$$set(array.mem, array.base\_addr + index, data)$$

Is this safe? No!

Proposed safe accesses and writes:

$$array.mem(array.base\_addr + (index \mod array.size)$$
$$set(array.mem, (array.base\_addr + (index \mod array.size, data)$$

Can we prove this? Yes!

Should you trust me/the system? NO! Let's figure out why it works.

Untyped Lambda Calculus

"*Because Schönfinkel has in no way shown how the introduction of the other fundamental concepts is to be avoided, and because he cannot define them from others, he has not justified his claim. In fact he has achieved only a new and inconvenient notation.*" - Haskell Curry

The Philosophy of Computation

- Lambda Calculus is a computing model that uses simple math definitions, instead of mechanical description of Turing Machines




The Philosophy of Computation

- Lambda Calculus is a computing model that uses simple math definitions, instead of mechanical description of Turing Machines
- LC/TM came about during a rough time in mathematics (1890-1930) when paradoxes had been found in our assumptions




The Philosophy of Computation

- Lambda Calculus is a computing model that uses simple math definitions, instead of mechanical description of Turing Machines
- LC/TM came about during a rough time in mathematics (1890-1930) when paradoxes had been found in our assumptions
- Wanted to make sense of what it meant to do computation, or represent an equation, or decide the truth value of a statement




Implementing the Lambda Calculus

$$\text{Syntax:} \quad e := v \mid \lambda v.e \mid (e_1)(e_2)$$

$$\text{Semantic(s?):} \quad \frac{}{(\lambda v.e_1)(e_2) \Rightarrow e_1[e_2/v]}$$

Implementing the Lambda Calculus

$$\text{Syntax:} \quad e := v \mid \lambda v.e \mid (e_1)(e_2)$$

$$\text{Semantic(s?):} \quad \frac{}{(\lambda v.e_1)(e_2) \Rightarrow e_1[e_2/v]}$$

That was...easy?

Implementing the Lambda Calculus

$$\text{Syntax:} \quad e := v \mid \lambda v.e \mid (e_1)(e_2)$$

$$\text{Semantic(s?):} \quad \frac{}{(\lambda v.e_1)(e_2) \Rightarrow e_1[e_2/v]}$$

That was...easy?

Maybe not very clear, let's try again

Implementing the Lambda Calculus

In LC we have “expressions”, which can be:

- Variable names (v)

We only need one rule to compute things: when applying two expressions, if the left is an abstraction, take the right expression and plug it into every occurrence of the abstraction’s variable in the abstraction’s expression.

Implementing the Lambda Calculus

In LC we have "expressions", which can be:

- Variable names (v)
- Functions with single arguments, a.k.a. abstractions ($\lambda v.e$)

We only need one rule to compute things: when applying two expressions, if the left is an abstraction, take the right expression and plug it into every occurrence of the abstraction's variable in the abstraction's expression.

Implementing the Lambda Calculus

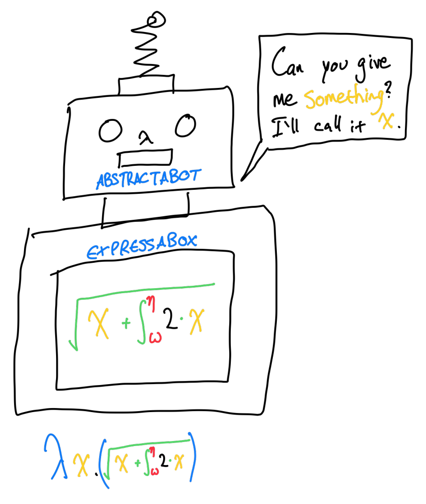
In LC we have "expressions", which can be:

- Variable names (v)
- Functions with single arguments, a.k.a. abstractions ($\lambda v.e$)
- Applications of two other expressions ($(e_1)(e_2)$)

We only need one rule to compute things: when applying two expressions, if the left is an abstraction, take the right expression and plug it into every occurrence of the abstraction's variable in the abstraction's expression.

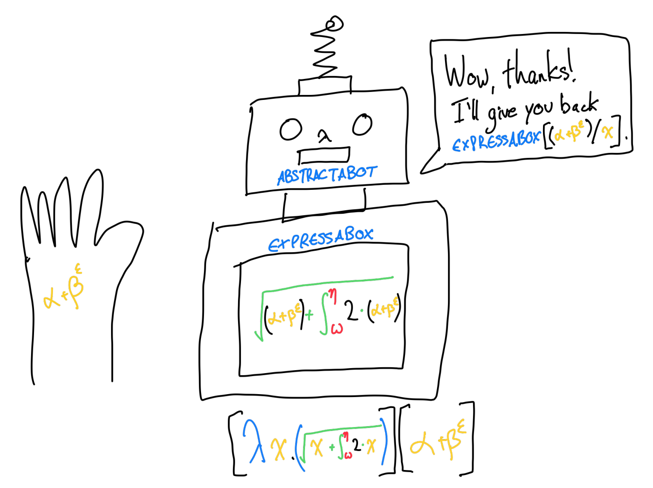
Implementing the Lambda Calculus

One more explanation for clarity:



Implementing the Lambda Calculus

One more explanation for clarity:



What can we do with LC?

Numbers:

$$0 := \lambda f.\lambda x.x$$
$$Successor := \lambda n.\lambda f.\lambda x.f(nfx)$$

Booleans:

$$True := \lambda x.\lambda y.x$$
$$False := \lambda x.\lambda y.y$$
$$if\ B\ then\ P\ else\ Q := \lambda B.\lambda P.\lambda Q.BPQ$$

Arbitrary loops: (try this for yourself $\ddot\smile$)

$$Y := \lambda f.(\lambda x.f(xx))(\lambda x.f(xx))$$

LC Issues

- No data types - ints can be used as bools and etc. and it's still a legal expression

How are we going to solve this?

LC Issues

- No data types - ints can be used as bools and etc. and it's still a legal expression
- Non-terminating expressions (huge wrench in the mathematics)

How are we going to solve this?

LC Issues

- No data types - ints can be used as bools and etc. and it's still a legal expression
- Non-terminating expressions (huge wrench in the mathematics)
- Partial evaluation is valid - makes debugging very difficult

How are we going to solve this?

Typed Lambda Calculus

"*For any formal system, we can really only understand its precise details after attempting to implement it.*" - Simon Thompson

Simply-Typed Lambda Calculus

As with many things in Computer Science, let's solve all of our problems by inventing Type Theory.

Simply-Typed Lambda Calculus

As with many things in Computer Science, let's solve all of our problems by inventing Type Theory.

We are going to add data types to LC and see where it takes us. We will gain things and lose things when we do this!

STLC

$$e := () \mid v \mid \lambda v : t.e \mid (e_1)(e_2)$$
$$t := \texttt{unit} \mid t_1 \to t_2$$

$$(1)\;\frac{}{(\lambda v : t.e_1)(e_2) \Downarrow e_1[e_2/v]}$$

$$(2)\;\frac{}{\texttt{typeof}(()) : \texttt{unit}}$$

$$(3)\;\frac{\texttt{typeof}(v) : t_1 \qquad \texttt{typeof}(e) : t_2}{\texttt{typeof}(\lambda v : t_1.e) : t_1 \to t_2}$$

$$(4)\;\frac{\texttt{typeof}(e_1) : t_1 \to t_2 \qquad \texttt{typeof}(e_2) : t_1}{\texttt{typeof}(e_1 e_2) : t_2}$$

Examples

$$\lambda x : \texttt{int}.\ x + 5$$

$$\lambda s : \texttt{string}.\ s\ \texttt{++}\ \text{“hello world”}$$

$$\lambda f : (\texttt{int} \to \texttt{int}).\ \lambda g : (\texttt{int} \to \texttt{int}).\ \lambda n : \texttt{int}.\ f(g(n))$$

Type Inhabitation

Another Theoretical Thing

- Let's take what seems to be a detour and think about a fun little puzzle.

Another Theoretical Thing

- Let's take what seems to be a detour and think about a fun little puzzle.
- First, we will add a new type to our STLC: `void`. `void` is special, because there is no value that has type void. Therefore, the type `void` is uninhabited.

Another Theoretical Thing

- Let's take what seems to be a detour and think about a fun little puzzle.
- First, we will add a new type to our STLC: `void`. `void` is special, because there is no value that has type void. Therefore, the type `void` is uninhabited.
- We know due to rule (2) that the expression () has type `unit`, so we say that "`unit` is inhabited by the value ()."

Another Theoretical Thing

- Let's take what seems to be a detour and think about a fun little puzzle.
- First, we will add a new type to our STLC: `void`. `void` is special, because there is no value that has type void. Therefore, the type `void` is uninhabited.
- We know due to rule (2) that the expression () has type `unit`, so we say that "`unit` is inhabited by the value ()."
- We can show that the type `unit` $\rightarrow$ `unit` is inhabited by the value:

Another Theoretical Thing

- Let's take what seems to be a detour and think about a fun little puzzle.
- First, we will add a new type to our STLC: `void`. `void` is special, because there is no value that has type void. Therefore, the type `void` is <u>uninhabited</u>.
- We know due to rule (2) that the expression () has type `unit`, so we say that "`unit` is <u>inhabited</u> by the value ()."
- We can show that the type `unit` $\rightarrow$ `unit` is inhabited by the value:

Another Theoretical Thing

- Let's take what seems to be a detour and think about a fun little puzzle.
- First, we will add a new type to our STLC: `void`. `void` is special, because there is no value that has type void. Therefore, the type `void` is <u>uninhabited</u>.
- We know due to rule (2) that the expression () has type `unit`, so we say that "`unit` is <u>inhabited</u> by the value ()."
- We can show that the type $\texttt{unit} \to \texttt{unit}$ is inhabited by the value:

$$\lambda x : \texttt{unit}.\ x$$

Another Theoretical Thing

So if `void` is uninhabited, and `unit` is inhabited, and `unit` $\rightarrow$ `unit` is inhabited, is this type inhabited?

$$\texttt{void} \rightarrow \texttt{unit}$$

Another Theoretical Thing

So if `void` is uninhabited, and `unit` is inhabited, and $\texttt{unit} \to \texttt{unit}$ is inhabited, is this type inhabited? Yes! $\lambda x : \texttt{void}.\ ()$

$$\texttt{void} \to \texttt{unit}$$

What about this type?

$$\texttt{void} \to \texttt{void}$$

Another Theoretical Thing

So if `void` is uninhabited, and `unit` is inhabited, and `unit` → `unit` is inhabited, is this type inhabited? Yes! $\lambda x : \texttt{void}.\ ()$

$$\texttt{void} \rightarrow \texttt{unit}$$

What about this type? Yes! $\lambda x : \texttt{void}.\ x$

$$\texttt{void} \rightarrow \texttt{void}$$

What about this type?

$$\texttt{unit} \rightarrow \texttt{void}$$

Another Theoretical Thing

So if $\texttt{void}$ is uninhabited, and $\texttt{unit}$ is inhabited, and $\texttt{unit} \to \texttt{unit}$ is inhabited, is this type inhabited? Yes! $\lambda x : \texttt{void}.\ ()$

$$\texttt{void} \to \texttt{unit}$$

What about this type? Yes! $\lambda x : \texttt{void}.\ x$

$$\texttt{void} \to \texttt{void}$$

What about this type? No!

$$\texttt{unit} \to \texttt{void}$$

More Types

Very quickly, let's add some more types:

- **Pairs**: (e_1, e_2). These expressions have type $\texttt{typeof}(e_1) * \texttt{typeof}(e_2)$, we call them "product types"

We can't make a pair that has a void in it, because no expression has type `void`.

More Types

Very quickly, let's add some more types:

- **Pairs**: (e_1, e_2). These expressions have type $\texttt{typeof}(e_1) * \texttt{typeof}(e_2)$, we call them "product types"
- **Constructed Types**: $\texttt{CON1}^{t_1+t_2}(e) \mid \texttt{CON2}^{t_1+t_2}(e)$. These expressions have type $t_1 + t_2$, we call them "sum types"

We can't make a pair that has a void in it, because no expression has type `void`.

More Types

Very quickly, let's add some more types:

- **Pairs**: (e_1, e_2). These expressions have type $\texttt{typeof}(e_1) * \texttt{typeof}(e_2)$, we call them "product types"
- **Constructed Types**: $\texttt{CON1}^{t_1+t_2}(e) \mid \texttt{CON2}^{t_1+t_2}(e)$. These expressions have type $t_1 + t_2$, we call them "sum types"

We can't make a pair that has a void in it, because no expression has type `void`.

More Types

Very quickly, let's add some more types:

- **Pairs**: (e_1, e_2). These expressions have type $\texttt{typeof}(e_1) * \texttt{typeof}(e_2)$, we call them "product types"
- **Constructed Types**: $\texttt{CON1}^{t_1+t_2}(e) \mid \texttt{CON2}^{t_1+t_2}(e)$. These expressions have type $t_1 + t_2$, we call them "sum types"

We can't make a pair that has a void in it, because no expression has type `void`.

But,

More Types

Very quickly, let's add some more types:

- **Pairs**: (e_1, e_2). These expressions have type $\texttt{typeof}(e_1) * \texttt{typeof}(e_2)$, we call them "product types"
- **Constructed Types**: $\texttt{CON1}^{t_1+t_2}(e) \mid \texttt{CON2}^{t_1+t_2}(e)$. These expressions have type $t_1 + t_2$, we call them "sum types"

We can't make a pair that has a void in it, because no expression has type `void`.

But, can we make a sum type with the signature $\texttt{unit} + \texttt{void}$?

More Types

Very quickly, let's add some more types:

- **Pairs**: (e_1, e_2). These expressions have type $\texttt{typeof}(e_1) * \texttt{typeof}(e_2)$, we call them "product types"
- **Constructed Types**: $\texttt{CON1}^{t_1+t_2}(e) \mid \texttt{CON2}^{t_1+t_2}(e)$. These expressions have type $t_1 + t_2$, we call them "sum types"

We can't make a pair that has a void in it, because no expression has type `void`.

But, can we make a sum type with the signature $\texttt{unit} + \texttt{void}$?

Yes! One value of this type is $\texttt{CON1}^{\texttt{unit}+\texttt{void}}(())$

More Types

Very quickly, let's add some more types:

- **Pairs**: (e_1, e_2). These expressions have type $\texttt{typeof}(e_1) * \texttt{typeof}(e_2)$, we call them "product types"
- **Constructed Types**: $\texttt{CON1}^{t_1+t_2}(e) \mid \texttt{CON2}^{t_1+t_2}(e)$. These expressions have type $t_1 + t_2$, we call them "sum types"

We can't make a pair that has a void in it, because no expression has type `void`.

But, can we make a sum type with the signature $\texttt{unit} + \texttt{void}$?

Yes! One value of this type is $\texttt{CON1}^{\texttt{unit+void}}(())$

Try showing that the following is inhabited:

$$\texttt{unit} * (\texttt{unit} \to (\texttt{void} + (\texttt{unit} \to \texttt{unit})))$$

Question Intermission

Binding Types with Logic

Type Checking

- Remember that we had some rules about the valid types of STLC expressions

$$(2)\ \frac{}{\texttt{typeof}(()) : \texttt{unit}}$$

$$(3)\ \frac{\texttt{typeof}(v) : t_1 \qquad \texttt{typeof}(e) : t_2}{\texttt{typeof}(\lambda v : t_1.e) : t_1 \to t_2}$$

$$(4)\ \frac{\texttt{typeof}(e_1) : t_1 \to t_2 \qquad \texttt{typeof}(e_2) : t_1}{\texttt{typeof}(e_1 e_2) : t_2}$$

Type Checking

- Remember that we had some rules about the valid types of STLC expressions

$$(2)\,\frac{}{\texttt{typeof}(()) : \texttt{unit}}$$

$$(3)\,\frac{\texttt{typeof}(v) : t_1 \qquad \texttt{typeof}(e) : t_2}{\texttt{typeof}(\lambda v : t_1.e) : t_1 \to t_2}$$

$$(4)\,\frac{\texttt{typeof}(e_1) : t_1 \to t_2 \qquad \texttt{typeof}(e_2) : t_1}{\texttt{typeof}(e_1 e_2) : t_2}$$

- The whole point of these is to be able to **statically** check the program (at compile-time) to ensure that it's well-typed (meaning we can't use numbers as booleans or strings as functions or etc.)

Type Checking

- Remember that we had some rules about the valid types of STLC expressions

$$(2)\ \frac{}{\texttt{typeof}(()) : \texttt{unit}}$$

$$(3)\ \frac{\texttt{typeof}(v) : t_1 \quad \texttt{typeof}(e) : t_2}{\texttt{typeof}(\lambda v : t_1 . e) : t_1 \to t_2}$$

$$(4)\ \frac{\texttt{typeof}(e_1) : t_1 \to t_2 \quad \texttt{typeof}(e_2) : t_1}{\texttt{typeof}(e_1 e_2) : t_2}$$

- The whole point of these is to be able to **statically** check the program (at compile-time) to ensure that it's well-typed (meaning we can't use numbers as booleans or strings as functions or etc.)
- Let's see how a simple type checker works

Curry-Howard Isomorphism

We're finally ready to assemble all of these pieces into a beautiful confluence between seemingly-unrelated things: The Curry-Howard Isomorphism.

Curry-Howard Isomorphism

We're finally ready to assemble all of these pieces into a beautiful confluence between seemingly-unrelated things: The Curry-Howard Isomorphism.

In short, it states that types are theorems, and programs are proofs of those theorems. Let's dig into why.

Why we did Type Inhabitation

The kinds of types we added to STLC were very deliberate:

Type	Inhabited
`unit`	True

Why we did Type Inhabitation

The kinds of types we added to STLC were very deliberate:

Type	Inhabited
`unit`	True
`void`	False

Why we did Type Inhabitation

The kinds of types we added to STLC were very deliberate:

Type	Inhabited
`unit`	True
`void`	False
`void * void`	False

Why we did Type Inhabitation

The kinds of types we added to STLC were very deliberate:

Type	Inhabited
`unit`	True
`void`	False
`void * void`	False
`void * unit`	False

Why we did Type Inhabitation

The kinds of types we added to STLC were very deliberate:

Type	Inhabited
unit	True
void	False
void * void	False
void * unit	False
unit * void	False

Why we did Type Inhabitation

The kinds of types we added to STLC were very deliberate:

Type	Inhabited
`unit`	True
`void`	False
`void * void`	False
`void * unit`	False
`unit * void`	False
`unit * unit`	True

Why we did Type Inhabitation

The kinds of types we added to STLC were very deliberate:

Type	Inhabited
`unit`	True
`void`	False
`void * void`	False
`void * unit`	False
`unit * void`	False
`unit * unit`	True
`void + void`	False

Why we did Type Inhabitation

The kinds of types we added to STLC were very deliberate:

Type	Inhabited
unit	True
void	False
void * void	False
void * unit	False
unit * void	False
unit * unit	True
void + void	False
void + unit	True

Why we did Type Inhabitation

The kinds of types we added to STLC were very deliberate:

Type	Inhabited
unit	True
void	False
void * void	False
void * unit	False
unit * void	False
unit * unit	True
void + void	False
void + unit	True
unit + void	True

Why we did Type Inhabitation

The kinds of types we added to STLC were very deliberate:

Type	Inhabited
unit	True
void	False
void * void	False
void * unit	False
unit * void	False
unit * unit	True
void + void	False
void + unit	True
unit + void	True
unit + unit	True

Why we did Type Inhabitation

The kinds of types we added to STLC were very deliberate:

Type	Inhabited
unit	True
void	False
void * void	False
void * unit	False
unit * void	False
unit * unit	True
void + void	False
void + unit	True
unit + void	True
unit + unit	True

Type	Inhabited
void $\rightarrow$ void	True

Why we did Type Inhabitation

The kinds of types we added to STLC were very deliberate:

Type	Inhabited
unit	True
void	False
void * void	False
void * unit	False
unit * void	False
unit * unit	True
void + void	False
void + unit	True
unit + void	True
unit + unit	True

Type	Inhabited
void $\to$ void	True
void $\to$ unit	True

Why we did Type Inhabitation

The kinds of types we added to STLC were very deliberate:

Type	Inhabited
unit	True
void	False
void * void	False
void * unit	False
unit * void	False
unit * unit	True
void + void	False
void + unit	True
unit + void	True
unit + unit	True

Type	Inhabited
void → void	True
void → unit	True
unit → void	False

Why we did Type Inhabitation

The kinds of types we added to STLC were very deliberate:

Type	Inhabited
unit	True
void	False
void * void	False
void * unit	False
unit * void	False
unit * unit	True
void + void	False
void + unit	True
unit + void	True
unit + unit	True

Type	Inhabited
void → void	True
void → unit	True
unit → void	False
unit → unit	True

Why we did Type Inhabitation

The kinds of types we added to STLC were very deliberate:

Logical Expression	Truth Value
T	True
F	False
$F \wedge F$	False
$F \wedge T$	False
$T \wedge F$	False
$T \wedge T$	True
$F \vee F$	False
$F \vee T$	True
$T \vee F$	True
$T \vee T$	True

Logical Expression	Truth Value
$F \to F$	True
$F \to T$	True
$T \to F$	False
$T \to T$	True

Why the CHI matters

Think about that - we have

- A language that natively encodes the philosophical ideas of theorems and proofs within its types and expressions

That means that with careful engineering, some type system extensions, and lots of confidence from lots of mathematicians, we can create a

high-assurance

Why the CHI matters

Think about that - we have

- A language that natively encodes the philosophical ideas of theorems and proofs within its types and expressions
- That can be (somewhat) trivially type-checked

That means that with careful engineering, some type system extensions, and lots of confidence from lots of mathematicians, we can create a

high-assurance

Why the CHI matters

Think about that - we have

- A language that natively encodes the philosophical ideas of theorems and proofs within its types and expressions
- That can be (somewhat) trivially type-checked
- Its semantics are simple enough that they fit on a single presentation slide

That means that with careful engineering, some type system extensions, and lots of confidence from lots of mathematicians, we can create a

high-assurance

Why the CHI matters

Think about that - we have

- A language that natively encodes the philosophical ideas of theorems and proofs within its types and expressions
- That can be (somewhat) trivially type-checked
- Its semantics are simple enough that they fit on a single presentation slide

That means that with careful engineering, some type system extensions, and lots of confidence from lots of mathematicians, we can create a

high-assurance

Why the CHI matters

Think about that - we have

- A language that natively encodes the philosophical ideas of theorems and proofs within its types and expressions
- That can be (somewhat) trivially type-checked
- Its semantics are simple enough that they fit on a single presentation slide

That means that with careful engineering, some type system extensions, and lots of confidence from lots of mathematicians, we can create a

high-assurance automated

Why the CHI matters

Think about that - we have

- A language that natively encodes the philosophical ideas of theorems and proofs within its types and expressions
- That can be (somewhat) trivially type-checked
- Its semantics are simple enough that they fit on a single presentation slide

That means that with careful engineering, some type system extensions, and lots of confidence from lots of mathematicians, we can create a

high-assurance automated proof checker.

Why the CHI matters

Think about that - we have

- A language that natively encodes the philosophical ideas of theorems and proofs within its types and expressions
- That can be (somewhat) trivially type-checked
- Its semantics are simple enough that they fit on a single presentation slide

That means that with careful engineering, some type system extensions, and lots of confidence from lots of mathematicians, we can create a

high-assurance automated proof checker.

Enter Rocq.

Rocq

"*Logic takes care of itself; all we have to do is to look and see how it does it.*" - Ludwig Wittgenstein

Rocq

- Rocq is an automated theorem proving system, containing a programming language called Gallina, as well as a proof language

Rocq

- Rocq is an automated theorem proving system, containing a programming language called Gallina, as well as a proof language
- Rocq is essentially an **implementation** of the Curry-Howard isomorphism, binding the concepts of types, theorems, programs, and proofs into a cohesive lambda calculus (CiC) that allows for high-assurance proof checking

Rocq

- Rocq is an automated theorem proving system, containing a programming language called Gallina, as well as a proof language
- Rocq is essentially an **implementation** of the Curry-Howard isomorphism, binding the concepts of types, theorems, programs, and proofs into a cohesive lambda calculus (CiC) that allows for high-assurance proof checking
- Has an extremely small TCB hand-verified by thousands of mathematicians for decades, and now machine-checked by projects implementing metatheory

The Calculus of Inductive Constructions

Rocq relies on the Calculus of Inductive Constructions, a variant of typed lambda calculus that combines the three primary type system additions, to provide the expressiveness necessary to state theorems that we're interested in:

- Parametric Polymorphism (type to term) - Adds a new kind of abstraction that takes a **type** as input and returns an expression ($\Lambda\alpha.e$) - allows us to express properties of generic types

The Calculus of Inductive Constructions

Rocq relies on the Calculus of Inductive Constructions, a variant of typed lambda calculus that combines the three primary type system additions, to provide the expressiveness necessary to state theorems that we're interested in:

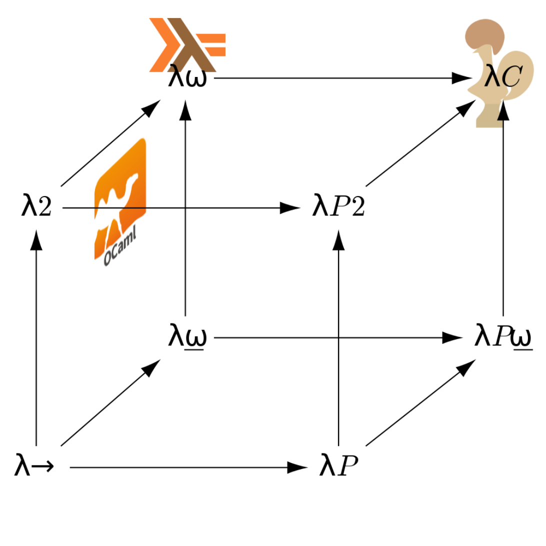
- Parametric Polymorphism (type to term) - Adds a new kind of abstraction that takes a **type** as input and returns an expression ($\Lambda\alpha.e$) - allows us to express properties of generic types
- Dependent Types (term to type) - Adds the capability to define expressions with types that change depending on the contents of the expression (e.g. `int list 5` vs `int list 3`) - this gives us the expressivity to define complex properties of quantified variables

The Calculus of Inductive Constructions

Rocq relies on the Calculus of Inductive Constructions, a variant of typed lambda calculus that combines the three primary type system additions, to provide the expressiveness necessary to state theorems that we're interested in:

- Parametric Polymorphism (type to term) - Adds a new kind of abstraction that takes a **type** as input and returns an expression ($\Lambda\alpha.e$) - allows us to express properties of generic types
- Dependent Types (term to type) - Adds the capability to define expressions with types that change depending on the contents of the expression (e.g. `int list 5` vs `int list 3`) - this gives us the expressivity to define complex properties of quantified variables
- Type Constructors (type to type) - Adds a new kind of abstraction that takes a **type** as input and returns a new type ($\Pi\alpha.t$) - this is necessary to avoid some paradoxes about the "type of types"

Lambda Cube



A Proof

```
Inductive nat : Type :=
| O
| S (n : nat).

Theorem add_0_r:
  forall (n : nat), n + 0 = n.
Proof.
  intros. induction n.
  (* if n = 0 *)
  - reflexivity.
  (* if n = S n' *)
  - simpl. rewrite IHn. reflexivity.
Qed.

Theorem andb_true: forall (b : bool), b && true = b.
Proof. intros. destruct b; reflexivity. Qed.
```

Breakdown

```
Inductive nat : Type :=
| O
| S (n : nat).
```

"There is a thing called 'nat' and it can either be O or it can be S applied to another nat."

Breakdown

```
Theorem add_0_r:
  forall (n : nat), n + 0 = n.
Proof.
  intros. induction n.
  (* if n = 0 *)
  - reflexivity.
  (* if n = S n' *)
  - simpl. rewrite IHn. reflexivity.
Qed.
```

“I propose this thing called ‘add_0_r’ which says that for all natural numbers n, $n + 0 = n$. I will prove it via an induction on n, using the inductive hypothesis in the inductive step.”

Breakdown

```
Theorem andb_true:
  forall (b : bool), b && true = b.
Proof.
  intros. destruct b; reflexivity.
```

"I propose this thing called 'andb_true' which says that for all booleans b, b && true = true. I will prove it via a case analysis of b."

Me: "I think this type is inhabited:"

$$\texttt{bool} : b \to (\texttt{eq}\ \ (\texttt{andb}\ \ b\ \ \texttt{true})\ b)$$

Rocq: "I don't believe you"

Me: "I'll show you it's inhabited:"

$$\lambda\, b : \texttt{bool}.\ \ \texttt{case}\ \ b\ \ \texttt{of}$$

$$\mid \texttt{false} \to \texttt{eq\_refl}\ \ (\texttt{andb false true})\ \ \texttt{false}$$

$$\mid \texttt{true} \to \texttt{eq\_refl}\ \ (\texttt{andb true true})\ \ \texttt{true}$$

What's the Point?

Rocq and other theorem provers are being used for tons of things:

- AWS and cryptography, general security, SAT solving

What's the Point?

Rocq and other theorem provers are being used for tons of things:

- AWS and cryptography, general security, SAT solving
- NASA and various mission-critical verified control systems

What's the Point?

Rocq and other theorem provers are being used for tons of things:

- AWS and cryptography, general security, SAT solving
- NASA and various mission-critical verified control systems
- Correct compilers

What's the Point?

Rocq and other theorem provers are being used for tons of things:

- AWS and cryptography, general security, SAT solving
- NASA and various mission-critical verified control systems
- Correct compilers
- One of various systems for verifying code for distributed systems

What's the Point?

Rocq and other theorem provers are being used for tons of things:

- AWS and cryptography, general security, SAT solving
- NASA and various mission-critical verified control systems
- Correct compilers
- One of various systems for verifying code for distributed systems
- Writing proofs about arbitrary machine code

What's the Point?

Rocq and other theorem provers are being used for tons of things:

- AWS and cryptography, general security, SAT solving
- NASA and various mission-critical verified control systems
- Correct compilers
- One of various systems for verifying code for distributed systems
- Writing proofs about arbitrary machine code
- And about a thousand other things

Conclusion

Summary

- We're not as good at guessing as we think we are

Want to learn? Check out Software Foundations, an incredible textbook designed to teach you the Rocq system from the ground-up.

Summary

- We're not as good at guessing as we think we are
- When lives are on the line, formal verification is one of the strongest methods to ensuring the correctness of software

Want to learn? Check out Software Foundations, an incredible textbook designed to teach you the Rocq system from the ground-up.

Summary

- We're not as good at guessing as we think we are
- When lives are on the line, formal verification is one of the strongest methods to ensuring the correctness of software
- FV is accomplished via elegant relationships between mathematics and programming

Want to learn? Check out Software Foundations, an incredible textbook designed to teach you the Rocq system from the ground-up.

Summary

- We're not as good at guessing as we think we are
- When lives are on the line, formal verification is one of the strongest methods to ensuring the correctness of software
- FV is accomplished via elegant relationships between mathematics and programming
- FV is on the rise - expect it to explode in popularity within the decade!

Want to learn? Check out Software Foundations, an incredible textbook designed to teach you the Rocq system from the ground-up.